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1) Specification of the game.

Actors are concerned about outcomes on a single issue-dimension represented by the real line $\Re$. Number veto players in such a way that $p_1 \leq p_2 \leq \ldots\ldots \leq p_i \leq \ldots\ldots \leq p_N$, where $p_i \in \Re$ is i's ideal point before any sidepayments are made and $N \geq 1$ is the number of veto players. Assume that there exists a veto i such that $p_i < 0$, where 0 is the status-quo policy.

Strategies:
Let k_{iJ} be the *expected* amount of capital that leader J allocates to veto i and $K_J \geq 0$ its total political capital. J's strategy consists of a pair $(\langle k_{1J}, k_{2J}, \ldots\ldots, k_{NJ}\rangle, x_J)$; a vector of capital allocation $\langle k_{1J}, k_{2J}, \ldots\ldots, k_{NJ}\rangle$, such that

$$\sum_{\forall i} |k_{iJ}| \leq K_J$$

and a policy proposal $x_j \in \Re$.

Veto i’s strategy is (v_{iA}, v_{iB}) where $v_{iJ} = 0$ if i rejects J's proposal and $v_{iJ} = 1$ if i accepts J's proposal.

Order of play:
Stage 1. A and B allocate political capital.
We consider three orders of play: i) B announces its allocation, then A; ii) allocations are announced simultaneously; iii) A announces its allocation then B.
Stage 2. A and B simultaneously announce policy proposals x_A and x_B.
Stage 3. Veto players simultaneously choose to accept or reject x_A and x_B.

Outcomes:
If $\sum_{\forall i} v_{iJ} = N$ then proposal x_J is accepted, else x_J is rejected.

Denote the outcome by Ω. If neither x_A nor x_B is accepted, $\Omega = 0$. If only x_j is accepted, $\Omega = x_j$, so long as $x_j \geq 0$, otherwise $\Omega = 0$. SUP{ . } denotes the supremum of an enumerated set. If x_A and x_B are both accepted, $\Omega = \mathrm{SUP}\{x_A, x_B\}$, so long as $\mathrm{SUP}\{x_A, x_B\} \geq 0$, otherwise $\Omega = 0$. Notice that $\Omega \geq 0$.

Payoffs:
$I_i = p_i + k_{iA} + k_{iB}$ where I_i is veto i's effective ideal point after lobbying. For any alternative x in the issue space, $u_i(x)' = -|I_i - x|$ where $u_i(\,.\,)'$ is veto i's utility function after lobbying. For leader J, $u_J(x) = -|I_J - x|$ where I_J is J's ideal point and $u_J(\,.\,)$ is its utility function. Assume that there exists a leader J for which $I_J > 0$.

Common Knowledge
Actors' utility functions, K_A, and K_B are common knowledge.

Equilibrium concept:
Sub-game perfect equilibrium.

Remark:
Stage 3 and the combination of stage 2 and stage 3 are proper sub-games. If stage 1 is simultaneous move, the only other sub-game is the game as a whole; if it is sequential there is also a proper sub-game starting with the move of the leader that lobbies second.

Lemma 1*:* Consider the veto strategy ν_i under which each veto i will accept any proposal x such that $-|I_i - x| \geq -|I_i - 0|$ i.e. with at least as high a payoff as the status-quo. The vector of veto strategies $\Upsilon = \{\nu_1, \nu_2, \ldots, \nu_I \ldots\nu_N\}$ constitutes an equilibrium in the sub-game starting in stage 3; and progress is maximised under this vector for any pair of proposals from leaders.

What any veto i does only makes a difference if a proposal will be the outcome so long as it does not veto it; and in this contingency it is as well not to veto if this leads to a payoff at least as high as the status-quo and otherwise to veto it. Hence Υ is an equilibrium of the sub-game. Moreover, no vector of veto strategies permit more progress than Υ. It must be the case that an equilibrium outcome in this sub-game is such that $\Omega \geq 0$, for the rules do not allow outcomes more conservative than the status-quo. Let INF{ . } denote the infimum of an enumerated set. Then it cannot be the case in an equilibrium that $\Omega > w = 2\mathrm{INF}\{I_1, I_2, \ldots I_N\}$: the outcome would be worse for at least one veto than the status-quo; and by switching its strategy, this veto could get an outcome at least as good as the status-quo. Progress is maximised when each veto accepts proposals at least as good as the status-quo, then. ÿ

Remarks.
i) When $\Omega > 0$ say the outcome is progressive. Then a necessary condition for a progressive equilibrium is $\mathrm{INF}\{I_1, I_2, \ldots I_N\} > 0$, or equivalently $w > 0$. Notice w is the right end of the winset of the status-quo when the vector of veto strategies is Υ.
ii) Innumerable equilibrium vectors of strategies for veto players generate less progress than Υ. For instance the vector in which all veto players block every proposal by voting against it is an equilibrium of the sub-game: for any individual veto, it makes no difference what it does because others will always block the proposal anyway. [1]

Lemma 2*:* Suppose that the vector of veto strategies is Υ. Then in any sub-game perfect equilibrium of sub-game constituted by stages 2 and 3 the outcome must be the same as if both leaders adopted the following proposal strategy, Π: if $w \leq 0$, the proposal $\pi = 0$; if $w > I_B$, $\pi = I_B$; if $0 < w \leq I_B$, $\pi = w$.

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[1] Because veto strategies that vote against proposals that are an improvement on the status-quo are weakly dominated by those that do so, they cannot be part of a *perfect* equilibrium; but they can be part of a sub-game perfect equilibrium.

Suppose that $w \leq 0$ i.e. there exists a veto i such $I_i \leq 0$. Then any pair of proposals can form part of an equilibrium: the outcome will always be the status-quo since i will veto any progressive proposal and the rules prevent proposals more conservative than the status-quo succeeding; hence the outcome is the same no matter what proposals are made.

Suppose that $w > I_B$. Say a proposal g governs the outcome if $g \leq w$, so it is not blocked by any veto, and $\Omega = g$. Under the rule that the most progressive proposal that is not subject to veto goes through, if there is a second proposal distinct from that governing the outcome, it must either be subject to veto or else it must be less progressive. Then it cannot simultaneously be true that there is a governing proposal $g > I_B$ and $w > I_B$ in any equilibrium. By assumption there is a veto i such that $p_i < 0$. For this Pareto inefficient outcome to be possible, A and B's dispositions of political capital to veto i would have to be such that enough net political capital is applied in a progressive direction to move i to a position such that $2I_i > I_B$. Suppose $K_A \geq K_B$. B can have expended no more that K_B on i in moving it to a position such that $2I_i > I_B$. Then A can transfer political capital to counter-lobby veto i at least as far as its original position p_i and, therefore, it is feasible for it to lobby i into a position where it will block progress beyond I_B. If $K_B > K_A$, by a similar argument B can change its allocation of capital so that i will block change beyond I_B, no matter how much of its capital A has expended to move i in a positive direction. So if $w > I_B$, then $g \leq I_B$ in any equilibrium. Suppose $g < I_B$. Then B can switch to proposing its own ideal point, none will veto, and the alternative proposal now governs the outcome. So in this case a proposal that governs the outcome in an equilibrium must be for I_B. Suppose no proposal governs the outcome. Then both proposals must be negative or subject to veto, so the outcome is the status-quo. But this cannot occur in equilibrium, because if $w > I_B$, B could then propose its ideal point. Hence in any equilibrium at least one leader must propose I_B and the other leader cannot propose something more progressive that is not subject to veto.

Suppose that $0 < w \leq I_B$. Then the proposal governing the outcome in any equilibrium must be for w. It cannot be for more than w because it would be vetoed. On the other hand if it was for less than w, B could propose w, which it prefers as an outcome. This proposal would not be subject to veto given the veto strategy vector Υ; and it would determine the outcome under the rule that the most progressive proposal that is not subject to veto goes through. Again there must be proposal governing the outcome; so in any equilibrium at least one leader must propose w.

We have shown that if the vector of veto strategies is Υ, in any equilibrium the outcome must be the same as if both leaders adopted the proposal strategy Π. There are other pairs of proposal strategies that could form part of an equilibrium; but none can give a different outcome since the governing proposal must be such that $\Omega = \pi$. When both leaders propose π their proposal strategies are in equilibrium given the veto strategy vector is Υ: where $w \leq 0$ it makes no difference what they propose; if $w > 0$, the outcome is any equilibrium is governed by π, so it makes no difference to its payoff if one leader switches strategy when the other proposes π. ÿ

Remark:
When the vector of veto strategies is Υ and the leaders' proposal strategies are Π, the degree of progress is equal to π.

2: Upper bounds on progress in sub-game perfect equilibria of the game.

By lemma 1, in each of the four cases considered in this section, maximum progress is possible in sub-game perfect equilibrium when the vector of veto players' strategies is Υ. Then by lemma 2 in any maximally progressive equilibrium, the outcome will be the same as if leaders both adopt the equilibrium proposal strategies Π and the degree of progress is π. Solving the overall game by backward induction, we now ask what lobbying strategies leaders will adopt in an allocation game at stage 1 knowing that they will adopt proposal strategies Π and that the vector of veto strategies is Υ. By characterising the equilibrium strategies in the allocation game, given both proposal strategies are Π and the vector of veto strategies is Υ, we can obtain the upper bound on progress in sub-game perfect equilibrium. Denote this upper bound by Φ.

Case 1*:* $I_A \le 0$ and $I_B > 0$

Because $\pi \in [0, I_B]$ by lemma 2, in the allocation game B wishes maximise π and A wishes to minimise π. Not only this, but B's payoff increases by the same amount that A's payoff falls for a given movement of π to the right on this interval, as a consequence of the form of leaders' utility functions. The allocation game is zero-sum, noting that there are no costs associated with using political capital. First we construct a Nash-equilibrium for the allocation game.

B's allocation strategy, alg_B, is defined by an algorithm. The algorithm terminates at the j-th iterations if no more of B's capital is unallocated or if $I_1' = I_2' = \ldots = I_j' = 0.5I_B + K_A$, where I_j' is equal to p_i plus the capital that B has allocated so far to veto i under the algorithm. If either of the conditions for termination holds say stop(j) is true. Then the algorithm is: 1) allocate capital to veto1 until $I_1' = p_2$ or stop(1) is true; 2) additionally allocate equal capital to veto players 1 and 2 until $I_1' = I_2' = p_3$ or stop(2) is true; ……..; j) additionally allocate equal capital to veto players 1, 2, ……,j until $I_1' = I_2' = \ldots = I_j' = p_{j+1}$ or stop(j)is true; ………..; N-1) additionally allocate equal capital to veto players 1, 2, ……,N-1 until $I_1' = I_2' = \ldots = I_{N-1}' = p_N$ or stop(N-1) is true; N) additionally allocate equal capital to all veto players until stop(N) is true. Given alg_B, A's strategy all_A is to pick at random one veto from the set of veto players $(v \mid I_v' = \mathrm{INF}\{I_1', I_2', \ldots I_N'\})$. Denote that veto by i. A spends all its political capital K_A counter-lobbying i to the position $I_i = I_i' - K_A$. Clearly all_A minimises π given B's strategy, since π is an increasing function of w. Given A's strategy, in order to maximise π, B must ensure that $\mathrm{INF}\{I_i', I_2', \ldots I_N'\}$ is as close as possible to

$0.5I_B+K_A$. The algorithm alg_B minimizes this distance within the constraint that B cannot allocate more than K_B: at each stage of operation of the algorithm the next resources allocated go those veto players in the most conservative positions — where they contribute most to minimizing the distance.

Because the allocation game is zero-sum, if it has Nash-equilibria, they will all have the same payoff vector as the one we have constructed (e.g. Owen, 1982, p.11). Because leaders' payoffs map one-to-one with the degree of progress, π, there will be the same degree of progress in any other Nash-equilibrium allocation. If the allocation game is simultaneous move, its sub-game perfect equilibria are the same as its Nash equilibria; if it is sequential move, they are a subset of the Nash equilibria. Either way the degree of progress is the same, because it is the same over all Nash-equilibria of the allocation game.

Denote the outcome of the game in case 1 when leaders' allocation strategies are all_A and alg_B; their proposal strategies are Π; and the vector of veto strategies is Υ by Ω_1. Then in case 1, $\Phi = \Omega_1$: most progress is possible if the vector of veto strategies is Υ (lemma 1); if the vector of veto strategies is Υ equilibrium proposals must give the same outcome as if both leaders adopted the proposal strategy Π (lemma 2); if the vector of veto strategies is Υ and proposal strategies give equivalent results to both leaders choosing Π, allocation strategies must give the same degree of progress as all_A and alg_B if they are to be in equilibrium, as we have just shown.

An expression for Φ when progress is made, implying $\pi \in (0, I_B]$, can be written down. Let the number of iterations after which alg_B terminates be r. Then

$$\Phi = p_r + \left(K_B - \sum_{i=1}^{r-1} (p_r - p_i)\right)/r - K_A$$

The way to see this is that it costs B $\sum_{i=1}^{r-1} (p_r - p_i)$ to move more conservative veto players to r's position leaving K_B minus this amount to be divided among r veto players to make further progress. Then A moves one of the furthest left veto players from the positions implied by B's allocation K_A to the left. Some algebraic manipulation gives

$$\Phi = \left(K_B - rK_A + \sum_{i=1}^{r} p_i\right)/r$$

Note that A's political capital is r times as effective in blocking progress than B's is in promoting it. In all cases $r \geq 1$ and typically r will be much larger than 1, as B will be able to lobby several veto players before running out of resources.

For $\pi = \Phi = 0$ it is necessary that

$$K_A \geq (K_B + \sum_{i=1}^{r} p_i)/r$$

Given relatively symmetric disposition of the initial positions of the veto players around the origin so that the summation term is prone to be no more than zero, the relatively weak sufficient condition for this is that $K_A \geq K_B/r$. By assumption at least one veto, say i, is such that $p_i < 0$. To get any progress after B's lobbying, all veto players (and therefore i) are in some position $x > 0$. This requires $K_B \geq (x - p_i)$; so if $K_A > K_B$, $K_A > (x - p_i)$, which means that A can counter-lobby i, moving it to a position where it will block any change. So if $K_A > K_B$, A can certainly block all change.

Denote the set of lobbies with initial positions to the left of I_B by S, with s members. To get $\Phi = I_B$, B must be able to lobby all members of S to a position K_A beyond its ideal point or

$$K_B \geq \sum_{i \in S}(I_B + K_A - p_i) = sK_A + \sum_{i \in S}(I_B - p_i)$$

That is K_B must be more than s times greater than K_A.

Definitions:

$\psi_B = 2\text{SUP}\{\text{INF}\{I_i', I_2', \ldots . I_N'\}, 0\}$ when B uses alg_B', which is exactly the same as alg_B except that the algorithm only ever stops allocating when B runs out of political capital. ψ_B is the upper limit on progress given A expends no capital and B uses its political capital to maximise the possible degree of progress. As we showed above alg_B pushes the right hand edge of the winset w as far to the right as possible given the vector of veto-players' strategies is Υ and A counter-lobbies one of the veto players furthest to the left under B's allocation. Similar arguments establish that if A allocates nothing, B maximises w under the same assumptions about veto players' strategies by using alg_B'. Notice that to maximise possible progress means to go on allocating resources even when B's ideal will not be subject to veto -- hence the modification to the stopping rule.

Now suppose that A's allocation rule is to use alg_A', which is just like alg_B' but starts from the vector of veto positions given when B's allocation is alg_B' i.e. $\{I_i', I_2', \ldots . I_N'\}$ when B uses alg_B'. $\Psi_{MAX} = 2\text{SUP}\{\text{INF}\{I_i, I_2, \ldots . I_N\}, 0\}$ when B uses alg_B' and then A uses alg_A'. If the vector of veto strategies is Υ, Ψ_{MAX} is the right hand end of the winset of the status-quo that could be achieved if both A and B lobby so as to maximise potential progress.

Case 2: $I_A > 0$ and $I_B > 0$ and $\Psi_{MAX} < I_A$

Here both A and B want more progress than they can possibly get. Maximal progress is possible when the vector of veto players' strategies is Υ (lemma 1) in which case equilibrium proposals must give the same degree of progress as if both leaders proposal strategy was Π (lemma 2); so possible progress is . an increasing function of w. A and B have a common interest in pushing w as far to the right as possible because, using the most efficient methods possible, equivalent to algA' and algB', they cannot get as much progress as they like, since $\Psi_{MAX} < I_A$. Hence in equilibrium in the allocation game they adopt strategies equivalent to algA' and algB' respectively: if $\Psi_{MAX} > 0$, this maximises their payoffs; if $\Psi_{MAX} \leq 0$, it makes no difference what strategy they adopt. Let the set of veto players with initial positions at or to the left of the status quo be denoted by C. Then a necessary condition for progress in this case is

$$K_A + K_B > \sum_{i \in C} - p_i$$

If progress is possible, the upper limit on progress is given by

$$\Phi \quad = \quad (K_B + K_A + \sum_{i=1}^{r} p_i)/r$$

where the r-th veto is the last one to which any political capital is allocated when A uses alg_A' and B uses alg_B'. Because A and B both want more progress than they can get acting jointly together, their political capital is equally efficacious.

Case 3: $I_A > 0$ and $I_B > 0$, $\Psi_{MAX} > I_A$ but $\Psi_B \leq I_A$.

By lemma 1 the vector of veto player's strategies allowing the greatest progress is Υ and, given these strategies, by lemma 2, leaders will use proposal strategies that give the same outcome as if both use Π. Suppose total allocations of political capital were such that $w < I_A$. Then as $\Psi_{MAX} > I_A$, the allocation strategies cannot be in equilibrium: as both leaders want more progress than this and they have the means to achieve it, one leader at least can change its allocation in such a way that w increases, leading to a better outcome. Suppose instead that $w > I_A$. Then it must be the case that A's allocation strategy is not a best reply to B's: if A allocates zero capital, w is at most $\Psi_B \leq I_A$; thus A must have adopted an allocation pushing w to the right of I_A, at variance with its interests, because this generates an outcome to the right of I_A . So if there exists a sub-game perfect equilibrium $w = I_A$. Suppose A uses algA" which is just like algA' except that it stops allocating political capital when $w = I_A$ and B uses algB'. Given that the vector of veto strategies is Υ so that both leaders use proposal strategies giving the same outcome as Π, these allocation strategies are in equilibrium: A cannot get a better outcome than its ideal point; no alternative to B's strategy can generate more of the progress B desires, because the most progress B can achieve if A allocates nothing is $\Psi_B \leq I_A < I_B$ and A's never pushes w to the right of I_A. The upper bound on progress is attained in this equilibrium, hence,$\Phi = I_A$. ÿ

Case 4: $I_A > 0$ and $I_B > 0$, $\Psi_{MAX} > I_A$ and $\Psi_B > I_A$.

By lemma 1 the most progress in a sub-game perfect equilibrium is possible when the vector of veto strategies is Υ in which case, by lemma 2, in any equilibrium the outcome is as if both leaders adopted the proposal strategy Π. In the allocation game, it cannot be the case that equilibrium allocations are such that $w < I_A$. Let V be the set of veto players such that for any veto v in V, $I_v = \text{INF}\{I_1 ,I_2,\ldots.I_N\}$. With leader B's allocation fixed ask the following questions about A's allocation strategy: q1) is any of A's political capital unallocated?; q2) is A allocating political capital to any veto v in V in a negative direction?; q3) is A allocating political capital to any veto i such that $I_i > I_v$? If the answer to any of these questions is yes, A's allocation is not a best reply to B's: A could allocate political capital to move the all members of V further to the right; and this will move the outcome nearer to I_A when leaders proposal strategies are Π and the vector of veto strategies is Υ. For example if political capital is being allocated to some veto outside V, some of that capital can be reallocated equally across members of V so their effective ideals move towards those of veto players on the right of them. Now given A's strategy, B's strategy cannot be a best reply if the answer to any of a similar set of questions to q1), q2) and q3) are true. Thus if $w < I_A$ and the allocation strategies of A and B are in equilibrium: both A and B are using all their political capital; no political capital is being allocated in a negative direction to any member of V; and no political capital is being allocated to veto players outside the set V. But this is a contradiction: when $K_A = 0$ the members of V must be at $0.5\Psi_B$ because B's allocation is exactly the same as under alg_B', moving members of V to a common position as far to the right as possible; as $\Psi_B > I_A$, the members of V must be further to the right than $0.5I_A$ if, in addition, A is spending political capital on them in a positive direction.

The argument then proceeds as in case 1. The allocation game is, in effect, zero sum: when the vector of veto strategies is Υ, the outcome will be as if leaders both adopt the proposal strategy Π, equilibrium allocations cannot give $w < I_A$, so leaders are playing an allocation game on the interval $[I_A, I_B]$ to get π as close as possible to their ideal point. As this allocation game is zero-sum, all equilibria have the same degree of progress. This is an upper bound on progress in the simultaneous move version of the allocation game in equilibrium -- hence on progress in the sequential move version of that game. The expression for this upper bound is exactly the same as that given in case 1. To get progress beyond I_A it is necessary that.

$$\Phi \quad = \quad (K_B - rK_A + \sum_{i=1}^{r} p_i)/r \;\geq I_A$$

or

$$K_B + \sum_{i=1}^{r} p_i \;\geq\; rK_A + rI_A$$

With a relatively symmetric initial distribution of veto players around the status-quo we can again argue that the summation term is likely to be no more than zero. Thus the condition for progress beyond A's ideal is likely to be a strict one. From the expression for Φ, A's resources are again r times as effective at blocking change as B's are in promoting it.

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